

Unique paper code : 62357502_OC

Name of the paper : Differential Equations

Name of the course: CBCS B.A.(Prog.)

Semester : V

Duration : 3 Hours

Maximum Marks : 75

Instructions for the candidate:

This question paper has six questions in all. Attempt any four. All questions carry equal marks.

1. Solve the initial value problem

$$(4x + y - 3)dy = (2x + 3y - 4)dx; y(1) = 1.$$

Solve $p^3(x + 2y) + 3p^2(x + y) + (y + 2x)p = 0$.

Find an integrating factor and solve the differential equation:

$$(e^{x+y} - y)dx + (xe^{x+y} + 1)dy = 0.$$

2. Find complete solution of the differential equation: $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \sin(2x + 3)$.

Consider the differential equation: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$. Show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solutions of the differential equation. Find the general solution. Also find the solution $y(x)$ with the property $y(0) = 2, y'(0) = -3$.

Solve $(2x + 3)^2 \frac{d^2y}{dx^2} - (2x + 3) \frac{dy}{dx} - 12y = 6x$.

3. Solve the differential equation using the method of variation of parameters:

$$y'' + 9y = 2\sec 3x.$$

Solve $(x^2D^2 - xD - 3)y = x^2 \log x$. (where differential operator $D = \frac{d}{dx}$)

Solve the following differential equation given that $y = e^x$ is an integral:

$$xy'' - (2x - 1)y' + (x - 1)y = 0.$$

4. Solve the following system of differential equation

$$\frac{dx}{dt} - 7x + y = 0, \frac{dy}{dt} - 2x - 5y = 0.$$

Solve $\frac{dx}{x^2yz(y^2-z^2)} = \frac{dy}{y} = \frac{dz}{z}$.

Solve:

$$zydx - zxdy - y^2dz = 0.$$

5. Eliminate the arbitrary function f from the equation

$$f(x + y + z, x^2 + y^2 - z^2) = 0$$

to find the corresponding partial differential equation.

Find the general solution of the differential equation

$$px(x + y) - qy(x + y) = -(x - y)(2x + 2y + 2z).$$

Find the complete integral of the partial differential equation

$$2xz - px^2 - 2qxy + pq = 0.$$

6. Classify the following partial differential equation into elliptic, parabolic or hyperbolic:

$$x^2(y - 1)r - x(y^2 - 1)s + y(y - 1)t + xyp - q = 0$$

where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

Form a partial differential equation by eliminating constants a, b from the relation:

$$ax^2 + by^2 + cz^2 = 1.$$

Find the general solution of the differential equation

$$x^2p + y^2q = x + y.$$

Find the complete integral of

$$px + qy + pq = z.$$